Soft gluon radiation at finite $N_c$ beyond leading order

Yazid Delenda (Université Batna 1) and Kamel Khelifa-Kerfa (Université Hassiba Benbouali de Chlef)

Prepared for the 1st Workshop on Matter and Radiation (WMR2016), April 10-11, 2016, Batna

**Motivation**

QCD precision measurements have seen significant improvements with the large integrated luminosity of the LHC. For instance, the strong coupling $\alpha_s$ has been determined by CMS to a 1.6% accuracy, with theoretical uncertainties affecting this measurement reaching up to 5% [1].

It is thus imperative that more precision is sought by theorists in order to boost the search for new physics, by tackling long-standing issues such as the delicacy of calculations of scattering amplitudes at higher orders in PT.

There are two problems which have long jeopardised calculations in QCD: (a) the complexity of the non-Abelian color structure, and (b) the factorially growing number of diagrams that need to be considered at each order. For these reasons a lot of calculations, including those which rely on Monte Carlo methods, resort to the large-$N$ approximation: number of quark colors) to reduce the complexity of the matrix-element structure. Furthermore, since typically threshold limits of QCD observables are the regions of the phase space that are most sensitive to contributions from higher orders in PT, one usually employs the Eikonal approximation which is sufficient to control up to single-logarithmic behaviour in the corresponding cross-sections.

With recent developments in computational physics, it is now in principle possible to overcome the above-mentioned issues and restore the finite-$N_c$ dependence of QCD scattering amplitudes.

**Objectives**

The aim of this work is to present a general formalism for the calculation of squared amplitudes of soft gluons at arbitrary orders in perturbation theory (PT) in QCD processes in the Eikonal approximation. For simplicity we choose the process $e^+ e^- \rightarrow q \bar{q}$, and demonstrate the calculation of the said squared amplitudes up to five loops.

**Eikonal Approximation at Leading Order**

Here we illustrate the trivial calculation at one loop (i.e. the process $e^+ e^- \rightarrow q \bar{q}$ shown in figure 2) which we later generalize at higher orders. The Eikonal approximation corresponds to the case where the emitted gluon is soft, i.e. its energy is much smaller than the centre-of-mass energy (i.e. scale of interaction). In this approximation the usual vertex/propagator Feynman rules can be simplified as shown in figure 3. Using these rules the one-gluon emission amplitude reads:

$$\sigma_1 = \sigma_0 \times 2 C_T g^2 \int \frac{d^4 k}{(2\pi)^2} \epsilon_k(k) \omega_\gamma(k) \left( \frac{p_+}{p_\mu} \cdot \frac{p_-}{p_\mu} \right) \cdot \epsilon^*_{(1) \mu}.$$

Figure 2: One gluon emission in $e^+ e^- \rightarrow q \bar{q}$.

**Virtual Corrections**

For virtual corrections, the only contributing diagram at one loop is that shown in figure 4. It turns out that the Eikonal squared amplitude, both for virtual and real emissions, factorizes into a product of a Born cross-section and a contribution from antenna functions. Furthermore, the virtual correction is simply given by minus the real emission contribution at this order:

$$\sigma_v = -\sigma_0 \times 2 C_T g^2 \int \frac{d^4 k}{(2\pi)^2} \omega_\gamma(k) \epsilon_k(k) .$$

Figure 4: One-loop virtual corrections.

**Examples**

First, let us define the following antenna functions:

$$A^{12}_{12} = \omega_2(\omega_2 + \omega_1 - \omega_3),$$

$$B^{12}_{13} = \omega_3(\omega_3 + \omega_1 - \omega_2).$$

The one and two-loops results are simple and well-known:

$$W^{12} = 2 C_T g^2 \omega_{12},$$

$$W^{13}_{12} = W^{12}_{13} = W^{12}_{13} + W^{13}{12},$$

where $A^{12}_{12}$ is a quark.

At two loops, the squared amplitude constitutes a sum of an independent emission term $W^{12}_{13}{12}$ (left diagram of figure 5) and an irreducible two-parton cascade emission term $A^{12}_{13}$ (right diagram):

$$\sigma_2 = \sigma_0 \times 2 C_T g^2 C_T A^{12}_{13} + B^{12}_{13},$$

with $A^{12}_{13} = \omega_3(\omega_3 + \omega_1 - \omega_2)$. The various contributions at this order are shown in figure 6.

Figure 5: Two-loops terms.

Figure 6: Three-loops terms.

Higher-order squared amplitudes which are more involved and too lengthy for presentation here are available in [4].

**EikAmp Program**

We have developed a Mathematica program called EikAmp, which automatically computes Eikonal squared amplitudes (theoretically at any order) by evaluating Eq. (1). The program spans over all possible Feynman diagrams through (or equivalently $\infty$) loops and, for each corresponding diagram, the color factor C is evaluated by calling the function CSimplify of the Package ColorMath [2]. The result is then interpreted in terms of a set of dipole antenna functions (which depend on the momenta of the outgoing partons) and a product of Casimir scalars ($C_T$ and $C_A$).

**References**


**Contact Information**

Y. D. Web: http://delenda.wordpress.com/

K. K-K. Web: http://khelifak.wordpress.com/